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Numerical Analysis of a Coaxial Line Terminated with a Complex Gap Capacitance

J. Obrzut and A. Anopchenko
National Institute of Standards and Technology
Gaithersburg, MD 20899, USA
Email: jan.obrzut@nist.gov

Abstract ***– A full wave numerical analysis was performed for a coaxial line terminated by a complex gap capacitance. The scattering parameters, input impedance and the spatial distribution of the electromagnetic field have been obtained in the frequency range of 100 MHz to 20 GHz for specimens 8 μm to 320 μm thick, with a the dielectric constant of up to 70. It was found that the impedance characteristic of the network is affected by the LC resonance coupled with the cavity resonance. Embedding in the network an inductive component, such as a section of the coaxial transmission line, allows one to decouple these two resonant behaviors. The specimen inductance is linearly dependent on the specimen thickness. At frequencies near the cavity resonance, the specimen section can be treated as a network of a transmission line with capacitance, where the fundamental mode propagates along the diameter of the specimen. The results are useful in improving accuracy of broadband dielectric measurements in extended frequency range of thin films with high dielectric constant that are of interest to bio-and nano-technology.***

I. INTRODUCTION

The relationship between the structure, function and stability of proteins and other biological substances has become an important research area of bio- and nano-technology. It is widely recognized that protein function depends on conformation, which dynamically changes with relaxation times in the sub-nanosecond range [1]. Microwave broadband dielectric measurements are well suited for studying conformational relaxation dynamics of such materials that typically exhibit a high dielectric constant, and are available in small quantities. Iskander and Stuchly proposed a refined technique for measuring the broadband dielectric properties of such materials using the reflection coefficient. As a sample holder, they used a small-gap shunt capacitor terminating a coaxial line [2]. This technique is accurate at frequencies where the sample holder can be treated as a lumped capacitance. They empirically determined that this technique is accurate up to a frequency at which input impedance of the specimen decreases to one tenth (0.1) of the characteristic impedance of the coaxial line [3]. The frequency limit is lower for thinner specimens that have a higher dielectric constant. For a 100 μm thick specimen with a dielectric constant of about 60, measured in the Amphenol Precision Connector 7 mm (APC-7) configuration, the upper frequency limit falls to within 85 MHz. This is well below the desirable frequency range of about 10 GHz to 20 GHz. Marcuvitz analyzed an equivalent circuit of a coaxial line, terminated by a small gap capacitance, treating it as a quasi-electrostatic problem. His analysis assumes a principal propagating mode in the coaxial line and no propagation in the gap [4]. Wave propagation has been observed experimentally in thin film specimens with a high dielectric constant [5]. Analysis of the fundamental mode in the specimen section was performed, treating the specimen as a network consisting of a transmission line with a capacitance. This led to a correction procedure that allowed extending the usable frequency range to about 8 GHz. A more fundamental analysis of the transverse electromagnetic wave scattering in a coaxial line terminated by a gap, was performed by Eom et al, using a Fourier transform and mode matching technique [6]. The results agreed with the Marcuvitz model up to frequencies of 12 GHz, but no satisfactory physical solution was obtained for gaps thinner than 100 μm and a dielectric constant larger than 10.

In this paper we present a full wave numerical analysis of the coaxial line terminated by a complex gap capacitance. The problem was formulated without any prior assumptions about the properties of the network equivalent circuit. The scattering parameters, input impedance and the spatial distribution of the electromagnetic field have been obtained in the frequency range of 100 MHz to 30 GHz for specimens 8 μm to 320 μm thick, with a dielectric constant of up to 70.

II. ANALYSIS

The three-dimensional field solution of Maxwell's equations was obtained using a finite element High Frequency Structure Simulator (HFSS) from Ansoft. The geometrical model of the network consisted of a 50 Ω 10 mm long APC-7 loss-less coaxial line. The diameter, *b*, of the line's outer conductor was 7.0 mm and the diameter, *a*, of the inner conductor was 3.0 mm. The specimen was represented by a dielectric gap of thickness, *d*, added to the central conductor and short terminated on the other side. The gap contained a dielectric having complex permittivity, $\varepsilon^* = \varepsilon' - j\varepsilon''$. The conducting surfaces at the dielectric were assumed to have the conductivity of copper. The reference plane was set at the beginning of the coaxial line. The geometric model was divided into approximately 40,000 tetrahedra elements as a finite element mesh. By representing the electric and magnetic field as a combination of finite quantities in each tetrahedron, Maxwell's equations were transformed into matrix equations that could be solved by traditional numerical methods. After calculating the electric and magnetic fields, the scattering parameters, S, of the network were obtained by comparing energy of the incoming and outgoing waves through the port located at the reference plane. The calculations were stopped when the change in the scattering parameters was less than 10^{-4}. The uncertainty in the S_{11} parameter was $\pm 10^{-4}$. The corresponding uncertainty of the impedance results normalized to 50 Ω, was $\pm 10^{-2}$ Ω.

III. RESULTS

The magnitude and phase of the S_{11} parameter, calculated in the frequency range of 100 MHz to 19 GHz for the 10 mm long coaxial line with a thin dielectric film specimen as a load, is shown in Fig. 1.

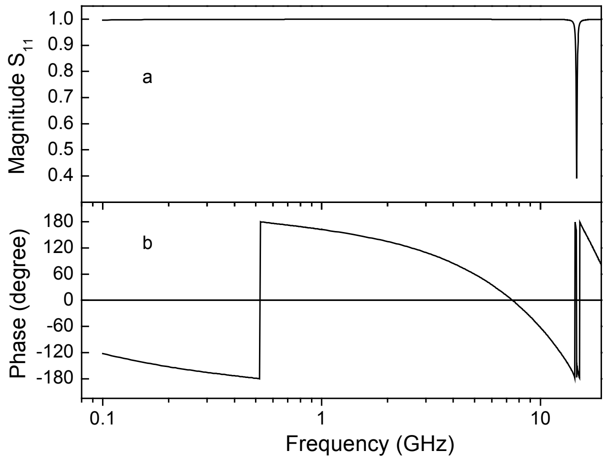

Fig. 1. Scattering parameter S_{11} at the port reference plane for a 10 mm long APC-7 coaxial line terminated with an 80 μm thick gap. The gap is filled with a dielectric of permittivity 69.2 - *j* 0.16; a: magnitude, b: phase.

The complex permittivity of the specimen is 69.2 - *j* 0.16 and its thickness is 80 μm. The magnitude plot of S_{11} shows a minimum at frequency, *f*, of 14.65 GHz. This feature originates from the primary cavity resonance in the specimen section. The 360 ° change in the phase shift at about 520 MHz indicates that the network exhibits LC series resonance at this frequency. Above 520 MHz the network's character changes from capacitive to inductive. This frequency agrees well with the resonant frequency calculated from the circuit parameters - the inductance, L_l, of the 10 mm long line, (L_l = 1.6946 nH) and the capacitance, C_s, of the specimen (C_s = 54.1 pF). Fig. 2.a shows a corresponding plot of impedance at the reference plane, where |Z| has a sharp minimum, resulting from LC resonance at 520 MHz. As the frequency increases, the phase shift of S_{11} approaches zero at about 7.8 GHz (Fig. 1.b) and the corresponding magnitude of impedance reaches a subsequent maximum (Fig. 2.a) at that frequency.

The effect of the transmission line can be de-embedded by transforming the scattering parameter matrix S_{11} from the port terminal to a new reference plane, located at the interface between the end of the center conductor and the specimen. The input impedance of the specimen, determined from the transformed S_{11}, is shown in Fig. 2.b. It is seen that the frequency of the LC resonance has shifted from 520 MHz to 5.05 GHz. Now, in effect, the LC resonance is coupled to the primary cavity resonance.

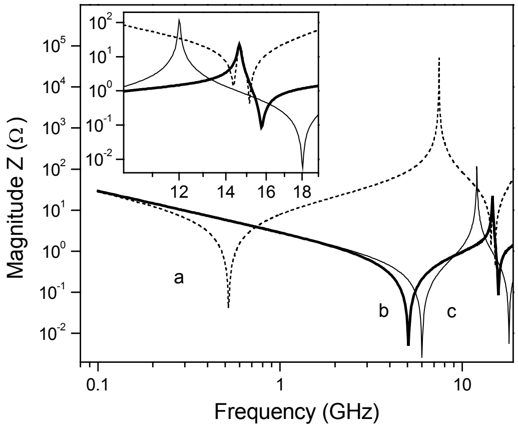

Fig. 2. Impedance vs. frequency. a: before de-embedding the coaxial line section, b: after de-embedding, c: calculated from equation (1). Details of the impedance characteristic near the cavity resonance are shown in the insert.

The cavity resonant frequency of 14.65 GHz remains unchanged by this transformation.

The input impedance of the specimen, calculated according to the model developed in [5], equation (1), is plotted in Fig. 2.c.

$$Z_{in} = \frac{x\cot(x)}{j\omega C_S} \tag{1}$$

Here C_s is the specimen capacitance, $\omega = 2\pi f$ is the angular frequency and x is the propagation factor, $x = \omega a/2c\,(\varepsilon^*)^{1/2}$. The specimen section in equation (1) represents a transmission line with a capacitance having propagation length of $a/2$ and no inductance. For the 80 μm thick specimen with complex permittivity of 69.2 – *j* 0.16, the expression (1) has a minimum at frequency of 6 GHz when $x = \pi/2$, followed by a maximum at 12 GHz, when $x = \pi$. The difference between the frequency, f_{LC}, of the first series resonance in Fig. 2.b at 5.05 GHz and that corresponding to the first minimum in Fig. 2.c at 6.0 GHz can be considered as resulting from the specimen residual inductance. The specimen inductance is included in the full wave numerical results plotted in Fig. 2.b, but neglected in the model represented by equation (1).

It has been commonly accepted that for thin film specimens the inductive component is negligibly small, and the electrical characteristic of the gap specimen is dominated by its complex capacitance. Most of analytical work on coaxial discontinuities applied this assumption to simplify the corresponding theoretical model. However, the results presented in Fig. 2.b indicate that the residual inductance of the specimen can considerably influence the impedance characteristic, especially at higher frequencies. In order to analyze this effect in more detail, we calculated the input impedance in relation to the film thickness, its diameter and the dielectric permittivity. The results are summarized in table 1. The table shows that, regardless of the film thickness, the f_{LC} remains unchanged for a given geometric configuration and permittivity.

Table1. Frequency of the series, f_{LC}, and cavity, f_{cav}, resonance in relation to the diameter, *a*, thickness, *d*, and permittivity, ε*, of the specimen.

a (mm)	*d* (μm)	ε^*	C_s (pF)	L_s (pH)	f_{LC} (GHz)	f_{cav} (GHz)
3 [APC-7]	8	34.6 - *j*1.6	271	1.73	7.35	20.15
3 [APC-7]	80	34.6 - *j*1.6	27.1	17.8	7.25	20.6
3 [APC-7]	320	34.6 - *j*1.6	6.77	72.1	7.2	20.6
6 [APC-14]	320	34.6 - *j*1.6	27.1	72.1	3.6	10.3
3 [APC-7]	80	69.2 - *j*1.6	54.2	18.3	5.05	14.65
3 [APC-7]	80	69.2 – *j*0.16	54.2	18.3	5.05	14.65

For example, in the APC-7 configuration, and at a permittivity of 34.6 - *j* 1.6, the f_{LC} is about 7.3 GHz as the specimen thickness increases from 8 μm to 320 μm. Therefore, the specimen inductance, L_s, can be determined from the value of the f_{LC}. In this example, when the specimen's thickness decreases from 80 μm to 8 μm, L_s decreases by a factor of 10. However, the specimen's geometric capacitance increases by the same factor. Thus, the resonant frequency, f_{LC}, which corresponds to the minimum value of the input impedance, remains unchanged. This finding indicates that the specimen inductance is linearly dependent on the specimen's thickness. Similar conclusion was arrived at by analyzing an equivalent circuit model of the circular waveguide filled with a dielectric material, that was recently proposed by Belhadj-Tahar [7]. In this new model, the specimen is represented by a capacitance in combination with an inductance. The solution of the equivalent circuit indicates that the specimen residual inductance can be expressed as a reciprocal function of the specimen geometric capacitance, which supports our results.

In addition to the circuit parameters, the high frequency impedance characteristic of the specimen is affected by the cavity resonance. The resonant frequency, f_{cav}, depends on the length of the resonator and the square root of the dielectric constant. To determine the actual resonator length, we analyzed the electric field associated with the fundamental mode propagating in the specimen. Fig. 3 shows the spatial distribution of the electric field magnitude inside the specimen at resonance. The magnitude of the electric field has radial symmetry with a maximum in the center of the specimen. In general, the magnitude of the electric field decreases along the radius, reaching a minimum at a distance that is 0.58 times the radius of the specimen. The fundamental mode propagates along the diameter of the specimen rather than along its thickness. In the APC-7 configuration where the diameter of the specimen is 3.0 mm, the distance between the adjacent minima of the electric field is approximately 1.74 mm, regardless of the dielectric permittivity. Thus, the effective size of the resonator is smaller than the diameter of the specimen.

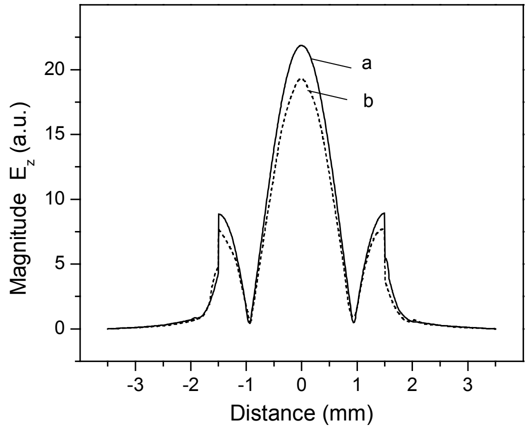

Fig. 3. Spatial distribution of the magnitude of the electric field E_z in the specimen section at the resonance; a: ε* = 34.6 - *j* 0.16, f_{cav} = 20.65 GHz, b: ε* = 69.2 - *j* 0.16, f_{cav} = 14.65 GHz.

Consequently the resonant frequency obtained by numerical analysis is higher than that estimated from equation (1), which assumes that the length of the resonator corresponds directly to the diameter of the specimen. As a result, the maximum value of the impedance presented in Fig. 2.b is shifted toward higher frequencies with respect to the corresponding maximum in Fig. 2.c.

IV. CONCLUSION

We have obtained a complete numerical solution for input impedance of a dielectric film specimen terminating a coaxial line. In contrast to the existing lumped capacitance approximations, we formulated the problem without prior assumptions about the network equivalent circuit.

The impedance characteristic of the network is affected by the LC resonance coupled with the cavity resonance. Embedding in the network an inductive component, such as a section of the coaxial transmission line, allows one to decouple these two resonant behaviors.

The residual inductance of the specimen is linearly dependent on the specimen thickness, and can be determined from the f_{LC} resonant frequency. At frequencies near the cavity resonance, the specimen section can be treated as a network of a transmission line with capacitance, where the fundamental mode propagates along the diameter of the specimen. If the specimen thickness is smaller than the diameter ($d < a$), the electrical length of the sample is 0.58 times the radius of the specimen and does not depend on the thickness.

We consider our analysis as a significant step towards improved understanding of wave propagation in coaxial discontinuities, particularly since, the distribution of electromagnetic field inside such structures is usually difficult to examine experimentally. Results of our analysis can be utilized to conduct more accurate broadband dielectric measurements of thin films with high dielectric constant at higher, microwave frequencies, that are of interest to bio-and nano-technology.

DISCLAIMER

Certain commercial materials and equipment are identified in this paper in order to adequately specify the experimental procedure and do not imply recommendation by the National Institute of Standards and Technology nor does it imply that the materials or procedures are the best for these purposes.

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